Numerical and experimental stress analysis of an external fixation system

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ABSTRACT
This paper presents research results of a stress analysis of a Sarafix external fixation system, applied to an unstable tibia fracture. A stress analysis was performed using Finite Element Analysis (FEA) and experimental stress measurements using strain gauges. Research was performed on the Sarafix external fixation system controlling values and directions of principal stresses at the measuring points in the case of axial compression. Sarafix proved to be mechanically stable, confirming good clinical results in the treatment of bone fractures.

Keywords: Computer Aided Design (CAD), Finite Element Analysis (FEA), experimental stress analysis, CATIA, principal stresses.

INTRODUCTION
After J.F. Malgaigne invented the external fixator in 1840, their selection and application was generally carried out on empirical grounds and accumulated experience in clinical orthopedics and traumatology [1]. In order to promote and carry out necessary research to improve fixation, a development of a theoretical analysis of problems fixation based on the principles of structural mechanics is pursued.

The external fixator is a medical device for the immobilization of fractures or serious damage to the structure of extremities. External fixation is a method of fracture immobilization achieved by the application of pins or wires into or through a bone and their binding to the outer frame. The above basic concept of the method has not changed since its origin, but progress is reflected through the development of new design solutions and materials used. In the last two decades, a closer link between medical science and other disciplines of science (Technics, Medical Engineering, Biomechanics etc.) has been created, with the aim of multidisciplinary solving contemporary medical problems. One example of association of scientists of different profiles for the purpose of designing and improving medical equipment is the application of methods of external fixation and the development of systems for external fixation.

The idea for the development of the external fixator Sarafix was developed by a group of orthopedists of “Prim. dr. Abdulah Nakati” General Hospital in Sarajevo under siege, in May 1992 [2]. The idea was triggered by the insufficient number of existing fixators, as the result of the expansion of the war activities in Bosnia and Herzegovina. Shortly after, the first fixator called Sarajevo war fixator - Sarafix (Figure 1) was produced. During the war, the Sarafix found its highest application in the treatment of extensive gunshot - explosive
fractures of long bones of the extremities. Today, in peacetime traumatology, it is used in accidental injury in traffic accidents and industrial trauma. Sarafix external fixation system represents a unilateral, biplanar external fixator which belongs to a group of modular fixators with one-half pins. Owing to the high flexibility and mobility, its application is possible to the complete human skeleton. Sarafix is the holder of numerous awards and prizes at international exhibitions of innovations, and gold medals at the exhibitions of innovations Brussels Eureka 95 and Geneva 1996, and Sarajevo’s Sixth of April Award for 2001 should be emphasized.

Objective and methods

All commercial fixators, now in use, passed a biomechanical study before their first application. Mechanical testing of Sarafix fixator was not performed before its clinical application, because of the war-time circumstances in which it originated. Complete mechanical research of the fixator, besides the examination of its stiffness to the loads to which it was exposed after the application, includes the analysis of stresses (von Mises and principal stresses) on the characteristic location of fixator design. Extensive studies of the mechanical research of the Sarafix fixator were carried out within the thesis [3]. Due to the limited scope of this paper, only the results of the axial compression tests will be presented.

With the aim of determining stability of external fixators, various sensors and transducers are set up on their designs [4]. During the past few years, except of performing the experimental testing, there has been an increased use of geometrical modeling and finite element analysis (FEA), in order to more fully describe the behavior of the fixator and its components during the loading [5].

This paper presents results of stress analysis of the most used configuration of the Sarafix external fixator in the case of an unstable tibial fracture. An open fracture at the middle of tibia with fracture gap of 50 mm (severe extensive injury with a considerable defect of bone structure) was examined. The most complicated aspect of bone fractures, both in terms of complexity of treatment and structural stresses of external fixator, is an open fracture. In the case of open fractures, in the initial phase of treatment, the full load is transferred through the fixator. The analyzed configuration of the Sarafix fixator contains four one-half pins in proximal and distal bone segment (Figures 1 and 2). The mechanical stability analysis of the Sarafix fixator was carried out using FEA and experimental analysis under axial compression.

Computer Aided Design (CAD) modeling of the Sarafix fixator and FEA were carried out at the Laboratory for Computer Aided Design - CADlab of the Mechanical Engineering Faculty Sarajevo. The first step consisted of forming a 3D geometrical model of the analyzed Sarafix fixator configuration, whereupon the FEA was performed on the model using CAD/CAM/CAE (Computer Aided Design/Computer Aided Manufacturing/Computer Aided Engineering) system CATIA [6, 7]. During the structural FEA, values of von Mises stresses were observed at two control points in the middle of the fixator connecting rod. The intensity and direction of principal stresses were monitored and analyzed at the same points.

Understanding the physical behavior of the model is a basic prerequisite for successful process of modeling real systems. Before that, it is necessary to make numerous assumptions related to modeling: structure, joints between the components, boundary conditions, loads, materials, etc. Fig. 2 shows the CAD and finite element model of the Sarafix fixator configuration.
The element method (FEM) model of the analyzed Sarafix fixator configuration after pre-processing. During the processes of the linear FEA, the material of wooden bone models was defined as orthotropic, while materials of the fixator design were modeled as isotropic. The FEM model consisted of solid finite elements of a linear (TE4) and parabolic tetrahedron (TE10) type. Joint elements of the spider type were used for modeling the joints between the components of the Sarafix fixator. The following joints were used: Fastened connection, Contact connection and Bolt tightening connection. The modeling of the influence of supports was performed using a Smooth virtual part. At the end of the proximal bone segment, the axial load in the form of surface force (Force density) was applied in the direction of the z axis of the Cartesian coordinate system. A displacement constraint of the Sarafix FEM model was derived by using the Ball join restraint on the model of distal bone segment. Likewise, a displacement constraint at the model of proximal bone segment was performed by using the User-defined restraint, which prevented the two translations in direction of x and y axis of the Cartesian coordinate system [8].

Experimental stress analysis was conducted at the Laboratory for materials testing and Laboratory for mechanical design testing of the Mechanical Engineering Faculty Sarajevo (Figure 3). At the Laboratory for materials testing, the examination of the analyzed configuration of the Sarafix fixator on the axial compression was performed, using a universal material testing machine (Zwick GmbH & Co., Ulm, Germany, model 143501). The analyzed configuration of the Sarafix fixator was attached to proximal and distal tibia bone segments modeled with cylindrical wooden bars with known physical properties. During the testing, the intensity of the load (0 to 600 N at the rate of 5 N/s) on the model of proximal segment of the tibia was controlled, using the force transducer (U2A, HBM-Hottinger Baldwin Messtechnik GmbH, Darmstadt, Germany). A wooden model of the proximal and distal bone segments are supported on the ball joint supports [9].

Tensometric measurement equipment (Laboratory for mechanical design testing) was used to control and monitor the value of the dominant principal stress on the two measurement points at the middle of the fixator connecting rod. The following equipment from the HBM manufacturer was used:

- digital measuring amplifier system (DMC) 9012A,
- computer with software for acquisition, monitoring and processing of measurement results – Catman, and
- four strain gauges (type 3/120LY11) connected in two Wheatstone half-bridges.

The strain gauges were placed on the opposite sides of the Sarafix fixator connecting rod at the same locations where intensities of maximum and minimum principal stresses were monitored during the FEA. Thereafter, the strain gauges were connected with the DMC system and computer through two separate channels. In this way, the maximum and minimum principal strains on the measuring points were measured independently [10].

This measurement method was applied because the connecting rod was subjected to a compound strain, which consisted of bending strain and axial compressive strain. The connecting rod, due to the axial compression at the proximal segment of the bone model, is exposed to the combined loading (eccentric pressure),
which consists of a combination of bending and axial compression. This form of the strain is manifested by
the unequal distribution of tensile and compression stresses along the longitudinal section of a connecting rod, i.e. neutral line does not coincide with the axis of symmetry of the fixator connecting rod. Therefore, the two separate Wheatstone half-bridges were formed and connected with the DMC system via two measurement channels. Wheatstone half-bridges consist of active strain gauge SG1 and compensation (inactive) strain gauge SG2 (Figure 3). The compensating strain gauges were placed near the active strain gauges on a plate tied to a connecting rod. Compensating strain gauges are used to compensate the effect of temperature on the measurement and they are of the same type as the active ones. The plate and connecting rod are made of the same material.

**Stress analysis**
The principal stresses of the stress tensor are the distinctive values of the stress tensor, while their direction vectors are the principal directions or eigenvectors [11]. When the coordinate system is chosen to coincide with the eigenvectors of the stress tensor, the stress tensor is represented by a diagonal matrix:

\[
\sigma = \begin{bmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_2 & 0 \\
0 & 0 & \sigma_3
\end{bmatrix}
\]  

where: \(\sigma_1, \sigma_2\), and \(\sigma_3\) are the principal stresses.

The values of the principal and von Mises stress were controlled on two locations at the middle of the fixator connecting rod during the FEA. The measuring point closer to the model of the bone segment was marked with MP- and the point on the opposite side of the connecting rod was marked with MP+ (Figure 4).

Compressive stresses, which were recorded at the measuring point MP- have a higher intensity compared to the tensile stress at the MP+. This is a direct consequence of the appearance of an eccentric compression that exposed fixator connecting rod. The direction of the maximum principal stress (\(\sigma_1\)) on the measuring point MP+ coincides with the direction of \(z\) axis, i.e. the axis of symmetry of the connecting rod. Likewise, the direction of the minimum principal stress (\(\sigma_3\)) on the MP- coincides with the axis of symmetry of the connecting rod. The minimum principal stress compared to the other two principal stresses at the MP- is dominant. Within the Fig. 4 a view B is given where directions and intensities of the principal stresses on the measuring points are presented. Note that at the MP+ the maximum principal stress is in fact the tensile stress, while at the MP- the minimum principal stress is actually the compressive stress. Also, it can be seen that the dominant principal stresses (\(\sigma_1\) and \(\sigma_3\)) are in the bending plane of the fixator which is not parallel with AP (anterior-posterior) plane. For this reason, the vectors of the dominant principal stresses do not match

![Figure 4. Plot of the principal stresses](image-url)
either (Figure 4, View B). Previously performed FEA determined the direction and intensity of the principal stresses. Also, it was noted that the intensities of the other two principal stresses at the measuring points were negligible compared to the maximum (σ₁ on MP+) and minimum (σ₃ on MP-) principal stress (Table 1). Active strain gauges are placed on the opposite sides of the connecting rod at the nearest and farthest point from the model of the bone, so that their longitudinal axis coincides with the directions of dominant principal strains (ε₁ and ε₃) at the measuring points [12]. The strain, registered by Wheatstone half-bridge with one active and one compensation strain gauge, is given by the relation:

\[
ε = \frac{4}{k} \frac{U_A}{U_E} \quad (2)
\]

where:
- \(k\) - is gauge factor,
- \(U_A\) - is bridge output voltage and \(U_E\) - is excitation voltage (bridge input) [13].

The dominant principal stresses at the measuring points (MP+ and MP-) are determined through the relations:

\[
σ₁ = ε₁ E \\
σ₃ = ε₃ E 
\]

Simultaneous measuring of the largest positive and negative principal strains on the opposite sides of the fixator connecting rod was carried out independently at two measurement points (Figure 5).

In the following analysis, the strain gauge placed on the side of the connecting rod closer to the bone model will be referred to as SG-, while a strain gauge placed on the opposite side will have a label SG+. This way of setting up strain gauges enables the measurement of the greatest positive principal strain (ε₁) at the measuring point MP+, on the basis of which the intensity of the maximum principal stress (σ₁) is determined. Analogously, on the measuring point MP-, the greatest negative principal strain (ε₃) was measured, on the basis of which the intensity of the minimum principal stress (σ₃) is determined. The minimum principal stress compared to the other two principal stresses at the point MP- is dominant.

Independently measured total strains at the measuring point consisted of the compressive and bending strain. The total (principal) strains are defined by the principle of superposition, as follows:

\[
ε₁ = -ε_p + ε₁ = \frac{-F}{AE} + \frac{M}{EZ} \\
ε₃ = -ε_p - ε₃ = \frac{-F}{AE} - \frac{M}{EZ} 
\]

where:
- \(ε_p\) - is the strain component caused by the axial compressive force,
- \(ε\) - the strain component caused by the bending moment,
- \(A\) - the area cross-section of the fixator connecting rod,
- \(E\) - modulus of elasticity,
- \(M\) - bending moment,
- \(Z\) - section modulus of the fixator connecting rod.

In this case of load, the bending strain was significantly higher than the compression strain (\(|ε_p|\gg ε\)). Distribution of the strains in the longitudinal section of the fixator connecting rod is shown schematically in the Fig. 5. Acquisition, display and processing of measurement results are performed using the HBM Catman software.

### Results

In order to achieve a direct comparison of results of the FEA and experimental analysis, all parameters of geometry, materials, loads, restraints on the FEM model are set according to experimental settings. Table 1 shows the intensities of principal and von Mises stresses at the measuring points.

<table>
<thead>
<tr>
<th>Location</th>
<th>Principal stresses</th>
<th>Methods</th>
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<tbody>
<tr>
<td></td>
<td>FEA, MPa</td>
<td>Exper., MPa</td>
</tr>
<tr>
<td>MP+, SG+</td>
<td>σ₁ 330</td>
<td>334</td>
</tr>
<tr>
<td></td>
<td>σ₂ 0,2</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>σ₃ 0,001</td>
<td>-</td>
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<tr>
<td></td>
<td>σ₁ -0,003</td>
<td>-</td>
</tr>
<tr>
<td>MP-, SG-</td>
<td>σ₂ -0,4</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>σ₃ -355</td>
<td>-368</td>
</tr>
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es stresses generated at the measuring points in the case of maximum axial compression force. The value of the maximum principal stress ($\sigma_1$) at the MP+ was significantly higher than the other two principal stresses ($\sigma_2$ and $\sigma_3$). Likewise, the value of the minimum principal stress ($\sigma_3$) at the MP- was significantly higher than the other two principal stresses ($\sigma_1$ and $\sigma_2$).

The maximum deviations of the results obtained by FEA in relation to the results obtained by experimental testing are range: the principal stress $\sigma_1$ to 1.2%, and the principal stress $\sigma_3$ to 3.6% (Figure 6).

Principal stresses with the negative sign represent compressive stress. It is noted that at the MP+ all principal stresses are positive, while at the MP- all principal stresses are negative (Table 1). The maximum values of maximum principal stress at the control points is $\sigma_3=368$ MPa and they are lower than the yield strength of the material of the fixator connecting rod ($\sigma_y=650$ MPa).

**Conclusion**

The conducted research has shown that there is a linear dependence between the loads and stresses generated on the connecting rod, as a result of the absence of large displacement and plastic deformation of the fixator components. The above fact is also a basic requirement for the fixator's stability in terms of preserving anatomical reduction of bone fragments in the postoperative load conditions. Detailed data of the stability of external fixation systems are needed by the orthopedic surgeon to predict successful healing of a fracture. The stability provided by the Sarafix fixator has been proven by mechanical research, confirming good clinical results in the treatment of bone fractures.

Comparing the results of FEA and experimental analysis of principal stresses at the measuring points reveals their good agreement. We can conclude that the developed FEM model of the Sarafix fixator was verified.

**DECLARATION OF INTEREST**

The authors declare no conflict of interest for this study.
References


